Probing Dark Energy Effects on Structure Formations

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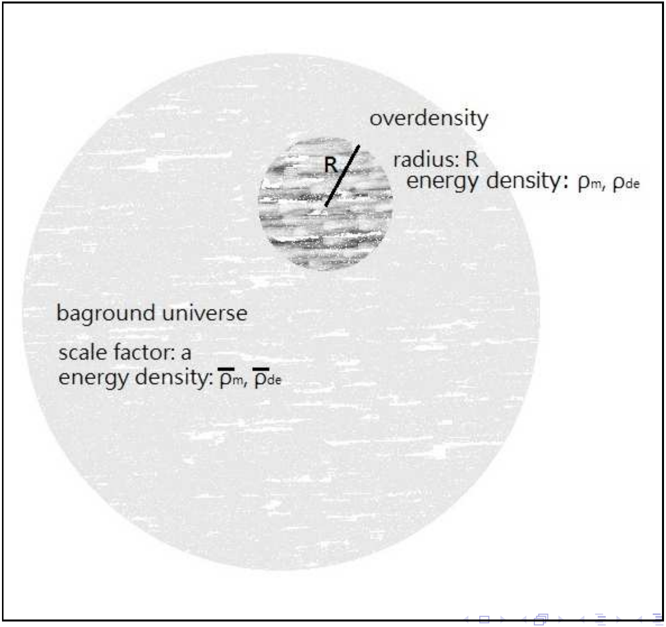
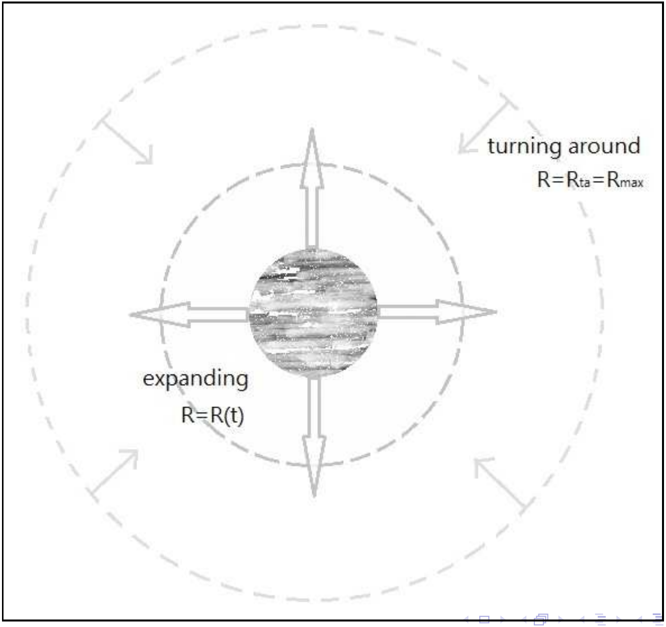
Outline

- The Spherical Collapse Model
- Virialization
- The Linear Evolution
- Comparison
- Conclusions

The Spherical Collapse Model



The Spherical Collapse Model



The Spherical Collapse Model

Background Evolution

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\bar{\rho}_{\rm m} + (1+3w)\bar{\rho}_{\rm de}\right]$$

$$\dot{\bar{\rho}}_{\rm m} + 3\left(\frac{\dot{a}}{a}\right)\bar{\rho}_{\rm m} = 0$$

$$\dot{\bar{\rho}}_{\rm de} + 3(1+w)\left(\frac{\dot{a}}{a}\right)\bar{\rho}_{\rm de} = 0$$

Spherical Perturbation

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left[\rho_{\rm m} + (1+3w)\rho_{\rm de}\right]$$

$$\dot{\rho}_{\rm m} + 3\left(\frac{\dot{R}}{R}\right)\rho_{\rm m} = 0$$

$$\dot{\rho}_{\rm de} + 3(1+w)\left(\frac{\dot{R}}{R}\right)\rho_{\rm de} = \alpha\Gamma,$$

where $\Gamma = 3(1+w)\left(\frac{\dot{R}}{R} - \frac{\dot{a}}{a}\right)\rho_{\rm de}$

with $0 \leq \alpha \leq 1$

The Spherical Collapse Model

clustering ($\alpha = 0$)

$$\frac{dx}{d\tau} = \sqrt{x^{-1} + \frac{1}{Q_{\rm ta}} x^{-3w-1}}$$

$$\Rightarrow \quad \tau_{\rm ta} = \frac{2}{3} F\left[\frac{1}{2}, -\frac{1}{2w}, 1 - \frac{1}{2w}, -Q_{\rm ta}^{-1}\right]$$

$$\frac{d^2 y}{d\tau^2} = -\frac{1}{2}\left[\zeta y^{-2} + (1+3w)\frac{1}{Q_{\rm cta}} y^{-3(1+w)+1}\right]$$

ζ : $\rho_{\rm m} =?$ $\Rightarrow y(\tau_{\rm c}) = y(2\tau_{\rm ta}) = 0$

$Q_{\rm cta}$: $\rho_{\rm de} =?$ $\Rightarrow$ *Assmuing* $\delta^{\rm NL}_{\rm de,ta} = r\delta^{\rm NL}_{\rm m,ta}$

$*d\tau \equiv H_{\rm ta}\sqrt{\Omega_{\rm m}(x_{\rm ta})}dt$, $x \equiv \frac{a}{a_{\rm ta}}$, $y \equiv \frac{R}{R_{\rm ta}}$, $\zeta \equiv \left.\frac{\rho_{\rm m}}{\bar{\rho}_{\rm m}}\right|_{z_{\rm ta}}$,

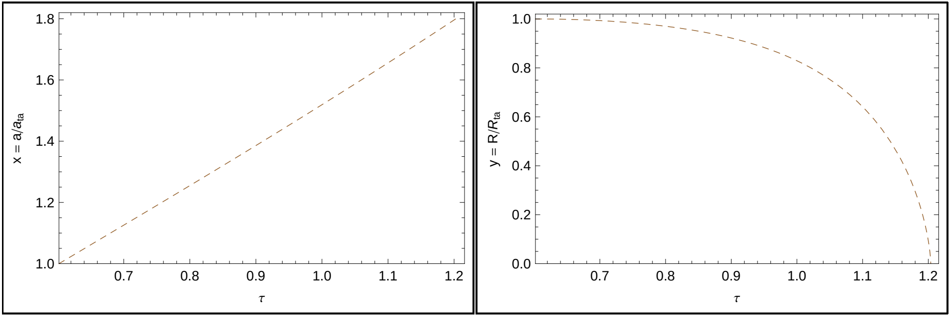
$Q_{\rm ta} \equiv \left.\frac{\bar{\rho}_{\rm m}}{\bar{\rho}_{\rm de}}\right|_{z_{\rm ta}} = \frac{\Omega^0_{\rm m}}{\Omega^0_{\rm de}}(1+z_{\rm ta})^{-3w}$, $Q_{\rm cta} \equiv \left.\frac{\bar{\rho}_{\rm m}}{\rho_{\rm de}}\right|_{z_{\rm ta}}$,

$\delta^{\rm NL}_{\rm m} \equiv \rho_{\rm m}/\bar{\rho}_{\rm m} - 1$, and $\delta^{\rm NL}_{\rm de} \equiv \rho_{\rm de}/\bar{\rho}_{\rm de} - 1$

The Spherical Collapse Model

clustering ($\alpha = 0$)

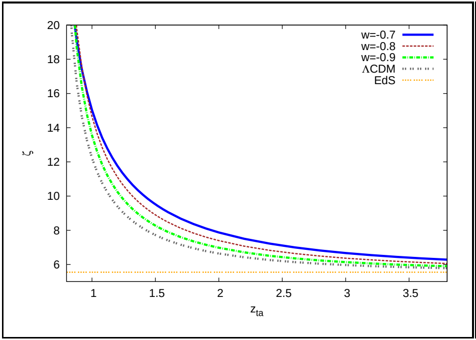
- The numerical solution of $dx/d\tau$ and $d^2y/d\tau^2$ for $w = -0.8$, $z_{\rm ta} = 0.8$ and $r = 1$:



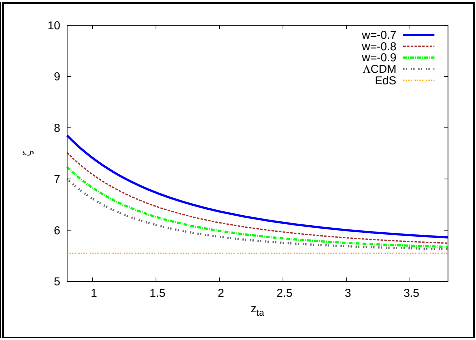
The Spherical Collapse Model

clustering ($\alpha = 0$)

- $r = 1$



- $r = 0.01$



$$* \; \zeta \equiv \left. \frac{\rho_{\rm m}}{\bar{\rho}_{\rm m}} \right|_{z_{\rm ta}}$$

Virialization

calculating the final size of the overdensity
refer to *JCAP* **07** (2005) 003

For the clustering case, the overdense region is regarded as an isolated system, so it holds for the virial theorem and the energy conservation.

- whole system virializing

$$[1+(2+3w)q+(1+3w)q^2]y_{\rm vir} \\ -\frac{1}{2}(2+3w)(1-3w)qy_{\rm vir}^{-3w}-\frac{1}{2}(1+3w)(1-6w)q^2y_{\rm vir}^{-6w}=\frac{1}{2}$$

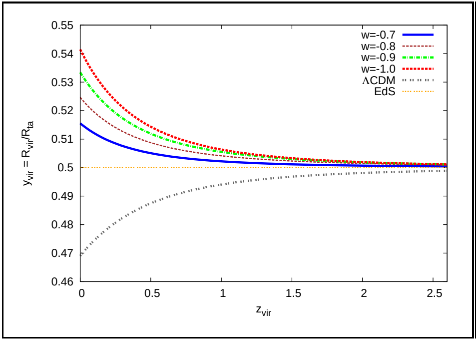
- only matter virializing

$$(1+q)y_{\rm vir}-\frac{q}{2}(1-3w)y_{\rm vir}^{-3w}=\frac{1}{2}$$

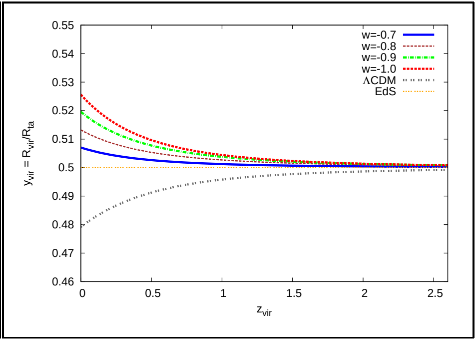
* $q \equiv \left.\frac{\rho_{\rm de}}{\rho_{\rm m}}\right|_{z_{\rm ta}} = \left(\frac{\delta^{\rm NL}_{\rm de,ta}+1}{\delta^{\rm NL}_{\rm m,ta}+1}\right)\frac{1-\Omega^0_{\rm m}}{\Omega^0_{\rm m}}(1+z_{\rm ta})^{3w}$
$= \left(\frac{r(\zeta-1)+1}{\zeta}\right)\frac{1-\Omega^0_{\rm m}}{\Omega^0_{\rm m}}(1+z_{\rm ta})^{3w}$

Virialization

- $r = 0.1$

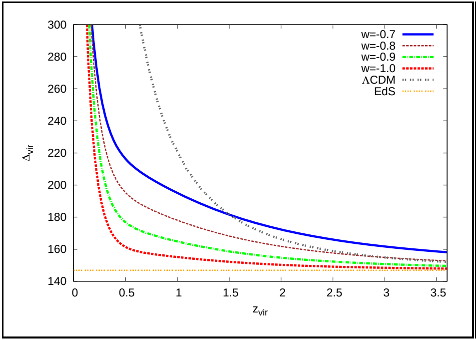


- $r = 0.01$

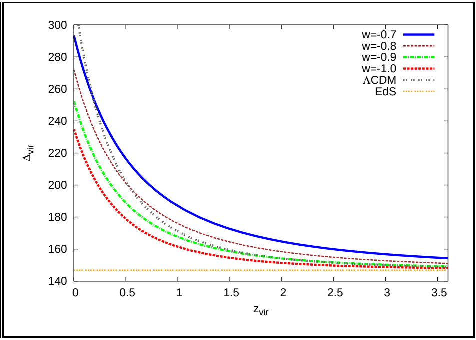


Virialization

- $r = 1$



- $r = 0.01$



$$* \; \Delta_{\rm vir} \equiv \left.\frac{\rho_{\rm m}}{\bar{\rho}_{\rm m}}\right|_{z_{\rm vir}} = \zeta \left(\frac{x_{\rm vir}}{y_{\rm vir}}\right)^3$$

The Linear Evolution

refer to Creminelli et al 2010 but for $\delta P_{\rm de} = w\delta\rho_{\rm de} = w\bar{\rho}_{\rm de}\delta_{\rm de}$

$$\frac{d^2\delta_{\rm m}}{d\eta^2} + \frac{2}{\tilde{x}}\frac{d\tilde{x}}{d\eta} = \frac{3}{2\tilde{x}^3}\left[\delta_{\rm m} + \frac{1-\Omega_{\rm m}^0}{\Omega_{\rm m}^0}(1+3w)\tilde{x}^{-3w}\delta_{\rm de}\right]$$

$$\frac{d\delta_{\rm de}}{d\eta} = (1+w)\frac{d\delta_{\rm m}}{d\eta} \Rightarrow \delta_{\rm de} = (1+w)\delta_{\rm m}$$

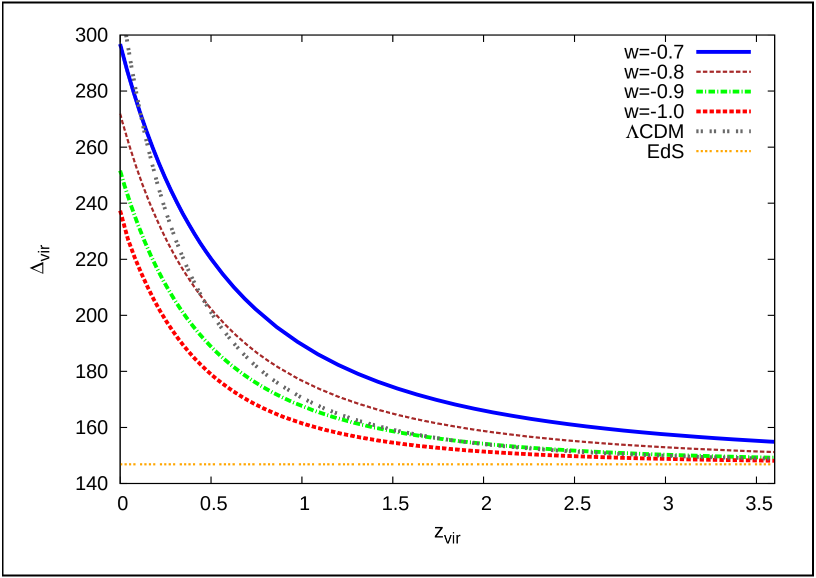
$$\frac{d\delta_{\rm m}^{\rm NL}}{d\eta} + 3(1+\delta_{\rm m}^{\rm NL})\left(\frac{1}{\tilde{y}}\frac{d\tilde{y}}{d\eta} - \frac{1}{\tilde{x}}\frac{d\tilde{x}}{d\eta}\right) = 0$$

$$\frac{d\delta_{\rm de}^{\rm NL}}{d\eta} + 3(1+w)(1+\delta_{\rm de}^{\rm NL})\left(\frac{1}{\tilde{y}}\frac{d\tilde{y}}{d\eta} - \frac{1}{\tilde{x}}\frac{d\tilde{x}}{d\eta}\right) = 0$$

$$\frac{d^2\tilde{y}}{d\eta^2} + \frac{1}{2}\left[\frac{1+\delta_{{\rm m},i}}{\tilde{x}_i^3}\frac{1}{\tilde{y}^2} + (1+3w)(1+\delta_{\rm de}^{\rm NL})\frac{1-\Omega_{\rm m}^0}{\Omega_{\rm m}^0}\frac{\tilde{y}}{\tilde{x}^{3(1+w)}}\right] = 0$$

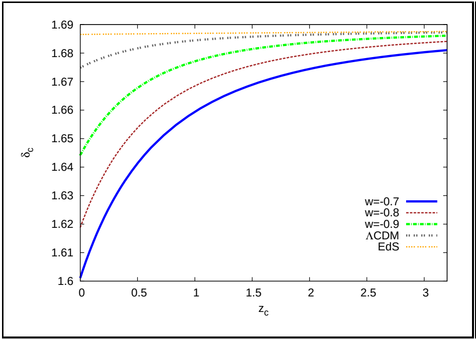
* $\tilde{x} \equiv \frac{a}{a_0}$, $\tilde{y} \equiv \frac{R}{R_i}$, $\eta \equiv \sqrt{\Omega_{\rm m}^0}H_0 t$, $\delta \equiv \rho/\bar{\rho} - 1$

The Linear Evolution

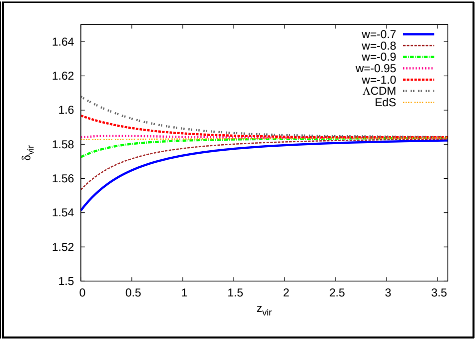


The Linear Evolution

- $\delta_{\rm m}$ as collapsing

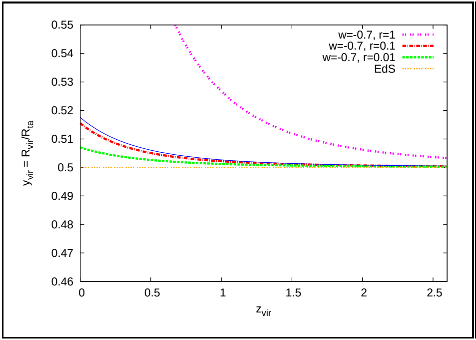


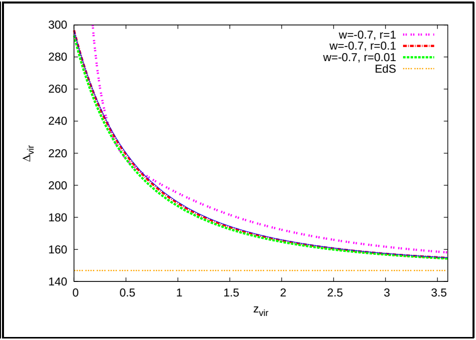
- $\delta_{\rm m}$ as virializing



Comparison

- $\delta^{\rm NL}_{\rm de,ta} = r\delta^{\rm NL}_{\rm m,ta}$
  $\Rightarrow$ trying different values of r
- making use of the approach in Creminelli et al 2010
  $\Rightarrow$ assuming the nonlinear density contrast of the spherical overdensity is linear at the very early time





Conclusions

- We solved the problem that the energy density of the clustering dark energy inside the overdense region is unknown by assuming $\delta^{\mathrm{NL}}_{\mathrm{de,ta}} = r\delta^{\mathrm{NL}}_{\mathrm{m,ta}}$.
- We studied the the evolution of the perturbation in the linear regime and assumed that the non-linear density contrast of the spherical overdensity is linear ($\delta^{\mathrm{NL}} \sim \delta$) at the very early time. It showed that we can find an appropiate value of r to mimic the result from this approach.
- It showed that the values of the obtained quantities converge to the one for the EdS universe at high reshift. The model dependence gets weak and the effect of the dark energy can be neglected in the early universe.